Research announcement from 1978:

AN INVARIANCE OF SYMMETRY

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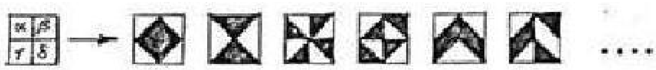
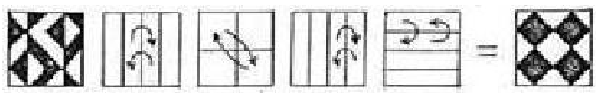
We present a simple, surprising, and beautiful combinatorial invariance of geometric symmetry, in an algebraic setting.

DEFINITION. A *delta transform* of a square array over a 4-set is any pattern obtained from the array by a 1-to-1 substitution of the four diagonally-divided two-color unit squares for the 4-set elements.

EXAMPLES. 

THEOREM. Every delta transform of the Klein group table has ordinary or color-interchange symmetry, and remains symmetric under the group G of 322,560 transformations generated by combining permutations of rows and columns with permutations of quadrants.

EXAMPLE. 

PROOF (Sketch). The Klein group is the additive group of GF(4); this suggests we regard the group's table T as a matrix over that field. So regarded, T is a linear combination of three (0,1)-matrices that indicate the locations, in T, of the 2-subsets of field elements. The structural symmetry of these matrices accounts for the symmetry of the delta transforms of T, and is invariant under G.

All delta transforms of the 4^5 matrices in the algebra generated by the images of T under G are symmetric; there are many such algebras.

THEOREM. If $1 \leq m \leq n^2 + 2$, there is an algebra of 4^m $2n \times 2n$ matrices over GF(4) with all delta transforms symmetric.

An induction proof contructs sets of basis matrices that yield the desired symmetry and ensure closure under multiplication.

REFERENCE

S. H. Cullinane, *Diamond theory* (preprint)