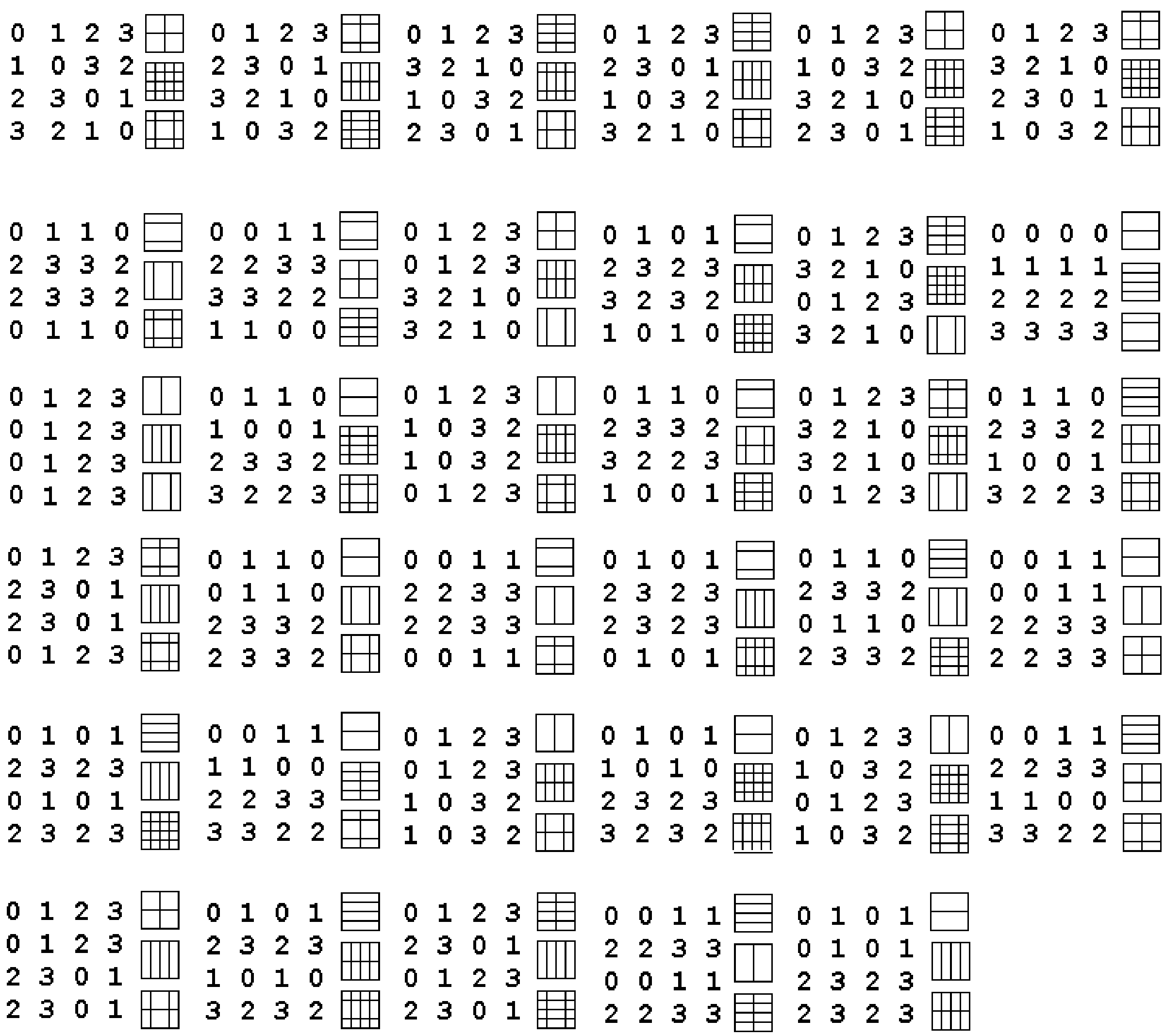
Steven H. Cullinane

Orthogonality of Latin squares viewed as skewness of lines. Dec. 1978.

Shown below is a way to embed the six order-4 Latin squares that have orthogonal Latin mates in a set of 35 arrays so that orthogonality in the set of arrays corresponds to skewness in the set of 35 lines of PG(3,2). Each array yields a 3-set of diagrams that show the lines separating complementary 2-subsets of {0,1,2,3}; each diagram is the symmetric difference of the other two. The 3-sets of diagrams correspond to the lines of PG(3,2). Two arrays are orthogonal iff their 3-sets of diagrams are disjoint, i.e. iff the corresponding lines of PG(3,2) are skew.

This is a new way of viewing orthogonality of Latin squares, quite different from their relationship to projective planes.

PROBLEM: To what extent can this result be generalized?



0 1 2 3	0 1 2 3	0 1 2 3	0 1 2 3	0 1 2 3	0 1 2 3
1 0 3 2	2 3 0 1	3 2 1 0	2 3 0 1	1 0 3 2	3 2 1 0
2 3 0 1	3 2 1 0	1 0 3 2	1 0 3 2	3 2 1 0	2 3 0 1
3 2 1 0	1 0 3 2	2 3 0 1	3 2 1 0	2 3 0 1	1 0 3 2

0 1 1 0	0 0 1 1	0 1 2 3	0 1 0 1	0 1 2 3	0 0 0 0
2 3 3 2	2 2 3 3	0 1 2 3	2 3 2 3	3 2 1 0	1 1 1 1
2 3 3 2	3 3 2 2	3 2 1 0	3 2 3 2	0 1 2 3	2 2 2 2
0 1 1 0	1 1 0 0	3 2 1 0	1 0 1 0	3 2 1 0	3 3 3 3

0 1 2 3	0 1 1 0	0 1 2 3	0 1 1 0	0 1 2 3	0 1 1 0
0 1 2 3	1 0 0 1	1 0 3 2	2 3 3 2	3 2 1 0	2 3 3 2
0 1 2 3	2 3 3 2	1 0 3 2	3 2 2 3	3 2 1 0	1 0 0 1
0 1 2 3	3 2 2 3	0 1 2 3	1 0 0 1	0 1 2 3	3 2 2 3

0 1 2 3	0 1 1 0	0 0 1 1	0 1 0 1	0 1 1 0	0 0 1 1
2 3 0 1	0 1 1 0	2 2 3 3	2 3 2 3	2 3 3 2	0 0 1 1
2 3 0 1	2 3 3 2	2 2 3 3	2 3 2 3	0 1 1 0	2 2 3 3
0 1 2 3	2 3 3 2	0 0 1 1	0 1 0 1	2 3 3 2	2 2 3 3

0 1 0 1	0 0 1 1	0 1 2 3	0 1 0 1	0 1 2 3	0 0 1 1
2 3 2 3	1 1 0 0	0 1 2 3	1 0 1 0	1 0 3 2	2 2 3 3
0 1 0 1	2 2 3 3	1 0 3 2	2 3 2 3	0 1 2 3	1 1 0 0
2 3 2 3	3 3 2 2	1 0 3 2	3 2 3 2	1 0 3 2	3 3 2 2

0 1 2 3	0 1 0 1	0 1 2 3	0 0 1 1	0 1 0 1
0 1 2 3	2 3 2 3	2 3 0 1	2 2 3 3	0 1 0 1
2 3 0 1	1 0 1 0	0 1 2 3	0 0 1 1	2 3 2 3
2 3 0 1	3 2 3 2	2 3 0 1	2 2 3 3	2 3 2 3